Economic Model for Trading Tokenized Investments

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Abstract

In this report we consider an Economic Model for Trading Tokenized Investments. We will limit ourselves to Tokens value tied firmly to current Investment Value. Later we apply this model to real estate and government bonds alike.

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We start with a company or an individual that starts with an investment. This is referred to "the company" throughout this report. We also build this model only for one investment and the implementation of it's tokenization.

1. Tokenization of Investment

The company comes forward and makes an investment. We take time as a variable starting from 0, going up to $T : t \in [0, T]$. The action of tokenization is at $t = 0$, with the fair market value of the investment being $P(t = 0)$. Here $P(t)$ *for* $t \in [0, T]$ is the value of the investment. For real estate, it is the market value of the land. For a bond its total value invested and accrued.

At time $t = 0$ this investment is alloted N number of tokens. The tokens are certificates with the following promises:

1. Tokens can be bought or sold to the company at the marketplace, always trading at (1/N) times the market value of the investment, $P(t)$ at any time,
2. Tokens can be traded among the users of the marketplace at any value (regulated by sorting by bids),
3. When the investment matures (or real estate land sold), for all Tokens, cash value equivalent to (1/N) times the sale value of the investment will be payed to the owner of the Token through a fast and safe method.

1.1 Sanity Check

From the Point of View of the company, we do a sanity test in the following way.

1. Company invests 100 Rs into property X
2. 5 years later property is worth 150 Rs, people buy half the tokens for 75 Rs
3. 5 years later property is worth 200 Rs, company sells and gains 200 Rs

4. Company returns half of 200 Rs to token owners

From the Point of View of the User of the marketplace, we do a sanity test in the following way.

1. Company invests 100 Rs and makes 20 tokens of it.
2. 5 years after company invests, the property is worth 150 Rs.
3. 10 Users buy 1 token each @ 7.5 Rs per Token, leading to the Users owning half of the total tokens
4. 5 years later property is worth 200 Rs. The company closes this investment and buys all the tokens back from the Users. 10 Users sell their tokens back to the company @ Rs (200 / 20) or Rs 10 each.
5. Each Token gives it's owner user a 2.5 Rs return on investment of 7.5 Rs.

Total User Revenue = −75 + 100 = 25 Rs. Profit Per User = 2.5 Rs for 7.5 Rs invested over 5 years. Total Company revenue = −100 + 75 + 200 − 100 = 75 Rs for 100 Rs invested over a period of 10 years. For the property, the appreciation in value is 100 Rs for investment of 100 Rs over a period of 10 years. Based on compound interest formula:

1. The Property gives an annual growth rate of 7.18%.
2. The Company gets an annual growth rate of 5.76%.
3. The Users each get annual growth rate of 5.92%.

1.2 Observations and Invariants

So far there is no assumption on what the underlying investment is. It can still be a property which can be sold at a profit, or any other investments. The following features of such a Trade can be observed:

1. The Company and the users end up sharing the profits from the appreciation of investment.
2. The Company can ensure that it always lands in profit by strategizing Token sale over time as long as the initial investment is in Profit.
3. The Users always make a profit as long as they buy the Tokens at a price less than the Token price when the investment is closed by the Company and the initial investment closes at a profit.
4. Even if no users end up buying any tokens, it is still an appreciating asset for the Company, giving safe return on investment.

2. Economic Model

2.1 Setup

It is time for some Mathematics. t is a variable in seconds, and is defined in the interval $[0, T]$, where investment is tokenized at time $t = 0$, and investment is closed (i.e. all the Tokens owned by Users are re-acquired by the Company through the marketplace at $T = 0$. As a feature of the Tokens, the company only trades them at the fair market value of the investment at any given time t, $P(t) : t \in [0, T]$. Thus the investment is tokenized at a price of $P(0)$ and closed/sold/matured at a price of $P(T)$.

Let $C(t) : t \in [0, T]$ be a function of time indicating the fraction of tokens owned by the Users at the marketplace at time t. Clearly this function is bounded by N, the total number of Tokens: $0 \leq C(t) \leq 1$ for $t \in [0, T]$. Then $C(t) = 1$ implies that all the tokens are owned by Users at the Marketplace, and all the proceeds coming from closure of investment will go the Token owners if time $t = T$. At the time of tokenization, all tokens are owned by the Company and made available for purchase via schemes Then $C(t) = 1$ implies that all the tokens are owned by Users at the Marketplace, and all the proceeds coming from closure of investment will go the Token owners if time $t = T$. At the time of tokenization, all tokens are owned by the Company and made available for purchase for the Users, thus $C(t = 0) = 0$.

Let us consider a Token trade by the Company at time t. We have an estimated investment price $P(t)$ and fraction of tokens owned by Users as $C(t)$ right before the trade. After the trade, $C(t)$ changes and goes a new value of $C(t + \Delta t)$.

Cash earned by the Company in this trade = $P(t) \times (C(t + \Delta t) - C(t))$

2.2 Measuring Revenue

Thus we can write an equation for the cash earned ($R(t)$) for the Company:

$$\begin{aligned}
\Delta R(t) &= P(t) \times (C(t + \Delta t) - C(t)) \\
\frac{\Delta R(t)}{\Delta t} &= P(t) \times \frac{(C(t + \Delta t) - C(t))}{\Delta t} \\
\frac{dR(t)}{dt} &= P(t) \lim_{\Delta t \to 0} \frac{C(t + \Delta t) - C(t)}{\Delta t} \\
\frac{dR(t)}{dt} &= P(t) \frac{dC(t)}{dt} \\
\int_{t=0}^{t} dt \, \frac{dR(t)}{dt} &= \int_{t=0}^{t} dt \, P(t) \frac{dC(t)}{dt} \\
R(t) &= R(0) + \int_{t=0}^{t} dt \, P(t) \frac{dC(t)}{dt}
\end{aligned}$$

Since the tokenization requires an initial investment value of $P(0)$, we can set $R(0) = -P(0)$:

$$R(t) = -P(0) + \int_{t=0}^{t} dt \, P(t) \frac{dC(t)}{dt} \tag{1}$$

Properties across India have been known to grow at roughly 10% throughout the period of 2011 - 2021. Let r be the rate of compounded year on year growth for the investment. At a 10% ($r = 1.1$) compounded growth rate, any investment doubles in value in around 7.27 years. It is important to note that r is also positive, $r > 0$; $0 < r < 1$ indicates an overall loss. $r > 1$ indicates $100 \times (r - 1)$ percent growth.

Then we approximate the value of our investment as $P(t) \approx P(0) r^{t/1Y}$. We can substitute this into equation 1:

$$R(t) = -P(0) + \int_{t=0}^{t} dt\, P(0) r^{\frac{t}{1Y}} \frac{dC(t)}{dt}$$

We look at the integral in the second term more closely:

$$\begin{aligned}
\int_{t=0}^{t} dt\, r^{\frac{t}{1Y}} \frac{dC(t)}{dt} &= \left[r^{\frac{t}{1Y}} C(t) \right]_0^t - \int_{t=0}^{t} dt\, C(t) \frac{dr^{\frac{t}{1Y}}}{dt} \\
&= r^{\frac{t}{1Y}} C(t) - r^{\frac{0}{1Y}} C(0) - \int_{t=0}^{t} dt\, C(t) \frac{dr^{\frac{t}{1Y}}}{dt} \\
&= r^{\frac{t}{1Y}} C(t) - \int_{t=0}^{t} dt\, C(t) \frac{dr^{\frac{t}{1Y}}}{dt} \; as\; C(0) = 0 \\
&= r^{\frac{t}{1Y}} C(t) - \int_{t=0}^{t} dt\, C(t) r^{\frac{t}{1Y}} \frac{ln(r)}{1Y} \\
&= r^{\frac{t}{1Y}} C(t) - \frac{ln(r)}{1Y} \int_{t=0}^{t} dt\, C(t) r^{\frac{t}{1Y}}
\end{aligned}$$

Substituting this back into our revenue equation:

$$\begin{aligned}
R(t) &= -P(0) + P(0) \left(r^{\frac{t}{1Y}} C(t) - \frac{ln(r)}{1Y} \int_{t=0}^{t} dt\, C(t) r^{\frac{t}{1Y}} \right) \\
&= -P(0) + P(0) r^{\frac{t}{1Y}} C(t) - \frac{ln(r)}{1Y} \int_{t=0}^{t} dt\, C(t) P(0) r^{\frac{t}{1Y}} \\
&= -P(0) + P(t) C(t) - \frac{ln(r)}{1Y} \int_{t=0}^{t} dt\, C(t) P(t) \\
&= P(t) C(t) - P(0) - \frac{ln(r)}{1Y} \int_{t=0}^{t} dt\, P(t) C(t)
\end{aligned}$$

Finally we add a term for closing the investment and gaining a revenue equal to $P(T)$:

$$R(t) = \quad = P(t) C(t) + P(T) - P(0) - \frac{ln(r)}{1Y} \int_{t=0}^{t} dt\, P(t) C(t) \tag{2}$$

2.3 Analysis

First part to note is that all the values in the final formula are positive except the logarithm. Generally, the *r* value of any appreciating asset is greater than 1, in which case this logarithm is also positive. However in case the investment runs into a loss,

the r value at that time would be between 0 and 1, leading to a negative logarithm, this making the last term positive.

There are several cases to be analyzed, each coming from a different $C(t)$ function, which is basically encodes consumer behaviour. The qualitative effect on the final total revenue is that the function $C(t)$ should be large (or close to 1), leading to a large positive first term $(P(t)C(t))$, and the area under the curve $C(t)$ should be small over $t \in [0, T]$ leading to a smaller integral which is the last term.

2.3.1 *Investing Revenue earned from Token Economy*

There is some scope of optimizing the return for the company from such a Tokenization Process. We modify our original example from the Section 1.1. From the Point of View of the company, we do a modified test in the following way:

1. Company invests 100 Rs into property X
2. 5 years later property is worth 150 Rs, people buy half the tokens for 75 Rs
3. Company takes half of the token revenue and invests further for the same rate of return as original investment.
4. 5 years later property is worth 200 Rs, company sells and gains 200 Rs
5. Company returns half of 200 Rs to token owners

Nothing changes from the Users Point of View. What changes is that the company earns an extra 7.18% return over a 5 year period due to ownership of Token revenue. Extra return earned is $(\frac{75}{2}) \times (1.0718)^5 = 53.0395 Rs$. The Total Company revenue increases by this amount and becomes Rs 128.0395 for an investment of 100 Rs over a period of 10 years. Based on the compound interest formula, this is an annual growth rate of 8.59%.

This is more than the original return rate of the investment. Overall, what has happened is that a company originally has made an investment of around 7% return. Then that company shares the return of that investment with people through the process of tokenization. And in turn increases the return to 8% for the same original investment.

2.3.2 *Inflation in Token Value when all the Tokens are owned by Users*

Let us now consider a case when Tokens are all owned by the Users. The company can no longer sell the Tokens at the marketplace for the agreed value of $(1/N)$ times the current market value of the investment. But Users can still try to sell the Tokens they own at the marketplace. Without the firm ground if the backing investment value, the Token value will start increasing because some Users will want to risk paying a slightly higher Token price in order to earn when the investment gets closed later on.

in such a scenario, the Token value will keep increasing unbounded, depending on the estimated value of the investment among the Users at the marketplace.

Lets take a solid example from earlier and follow a chain of such Users:

- Company opens an investment of Rs 100
- 5 years later the property is worth Rs 150

- The company sells all N=150 tokens to the Users at the marketplace for total Rs 150 (1 Token = 1 Re)
- 5 years later the investment becomes worth 200 Rs.
- a user purchases 1 Token @ 10% inflation => 1 Token = Rs 1.467 instead of Rs 1.333

Now there are two cases, one where company closes the investment when the estimated token value by the user (ETV) is more than market token value ($MTV = \frac{InvestmentMarketValue}{N}$). The company only returns the MTV, but user expected to get ETV based on their investment. Thus the user looses the Token Value to inflation. This value actually goes into the User who sells their Tokens at an inflated price. Besides the transaction fee, the only effect of such inflation has on company revenue is that the total value flowing in the marketplace temporarily increases. At 10% inflation, an additional 0.133 Rs were present in the marketplace than otherwise.

In the other case, the market value of the investment is greater than the value user paid for the Token times N. Then the user doesnt run into a loss, but their return is partially eaten up by inflation. Again the user who sells their Token at a bargain for profit are the ones who actually earn. This also has the same effect in marketplace capital flow and company revenue as the former case.

2.3.3 Inflation in Token Value when investment value is inflated

When the market value of an investment is inflated, the value of the tokens representing that investment will also inflate. This is because the tokens are essentially a claim on a share of the investment, and as the value of the investment increases, so too does the value of the tokens.

For example, let's say that a company invests Rs 100 in a real estate project. The company then tokenizes the investment, creating 100. Each token represents a claim on a share of the real estate project, and is therefore worth Re 1. Now, let's say that the market value of the real estate project increases to Rs 200. This means that each token is now worth Rs 2, because each token still represents a claim on a share of the same real estate project. This inflation in token value can be beneficial for investors, as it allows them to profit from the appreciation of the underlying investment. However, it is important to note that token value can also decrease if the market value of the underlying investment decreases.

The real shocker happens when the investment is closed but its estimated market value was inflated. This means that the investment sells for a smaller value than estimated, meaning that the Token value was inflated due to an inflated estimation of the market value of the investment itself. If the final investment closes on 150 Rs instead of the 200 Rs originally estimated, the Users who own the tokens will get a smaller overall return. Some will run into losses because they might have purchased the property after the market value of the investment is greater than final closure value, ie. all the Users who purchased such Tokens from the company when investment value was more than Rs 150 will now run into a loss as the investment finally always gets closed at Rs 150. This means that the company has an incentive to estimate the investment prices as higher than they currently are and close the investments at a loss. Since profits are shared when investments are closed, so are losses, allowing the

company to recuperate its losses from the revenue coming from Users purchasing tokens on the Marketplace. The company still earns a profit because the investment value has appreciated although lesser than estimated.

The higher the loss for the Users, the higher are company profits. Inflation of this kind benefits the tokenizing company only since the money lost by a User on Token value in this case goes directly to the company.

2.3.4 Token Value Self Correction when Investment is closed

In pure supply demand terms, we can interpret the final closure of investment by the company as a market correction in it's Token values to the fair market value of the investment.

To understand this, we can take an example. Lets suppose yet again, the market value of an investment starts at 100 Rs and linearly increases to 200 Rs in 10 years. However due to some behind the curtain reasons, the estimated value of the investment is inflated, so at around 5 years after initial tokenization, let us say that the market value of the investment which was supposed to be Rs 150 was inflated and estimated to be 200 Rs.

Since the property is appreciating, Users might still be interested in purchasing Tokens, thereby leading to increase in Token value, leading to an increase in estimated value of the investment. Some people might purchase Tokens at Rs 3 each which means they will incur a loss of Rs 1 per Token when the investment is closed at Rs 200 total. However, when the investment is closed, the company buys all the Tokens back only at the promised price of Rs 2 per Token, thus leading to a loss for all the Users that purchased the Tokens at Rs 3. Thus we can say that the market value of the Tokens undergoes a correction, settling down tending towards the fair market value of the investment. Anyone who purchased Tokens for more than this final corrected value takes a loss, anyone who purchased the Tokens at a price less than this corrected value makes a profit.

2.4 Conclusion

Let us consider the ways a company earns revenue in such a tokenization of an investment:

- The Revenue Earned from Token Purchase
- The Revenue Earned from Re-investing Token Revenue
- The Revenue Earned from Sharing Return from original Tokenized Investment

The first case is related to inflation of Token value. The more is the Token value when a User purchases it and the lower it is when the investment is closed, the higher is company revenue due to User loss. This points towards a scam economy as not only can the company artificially inflate the Token value since it is based on an estimate of the market value of the investment, but also the company can close the investment at a smaller value by unfair means. There is little mechanism in a simple process of tokenization to give fairness guarantees from the companies side. In order to fully be fair, the company must guarantee to keep the reported investment value close to the fair market value and it should be verifiable by a third party.

The second point is just a simple re-investment of acquired revenue. This helps to an extent, but a company would likely need more than just this to keep itself afloat when it comes to a primary source of revenue.

The third and last point is simply a sharing of return from the tokenized investment. In this case, the more people buy the Tokens, the less company earns and the more Users will earn. However the total return for both parties will be less than the return if they invest the same value in a similar investment individually. This sharing might be attractive for Users for short term investments, but in that case the company will simply be trading off its return from investment just to sell tokens. It might be safer and simpler for a company to simply make an investment and not tokenize it if it depends only on revenue from the third point.

Overall, the process of tokenization in the simple manner described here is simply not enough for a company to firmly secure it's revenues. The company needs to be doing more than just sharing to generate value for the society. If a simple tokenization process such as the one considered here is advertised as the main revenue stream for a company, it is likely to be scam and Users should stay away unless met with convincing evidence.