# Indian Institute of Technology Hyderabad

## Angular Correlations in $pp \to t\bar{t}$

Abhishek Agarwal
EP17BTECH11001

*Under the guidance of Professor Priyotosh Bandhyopadhyay*

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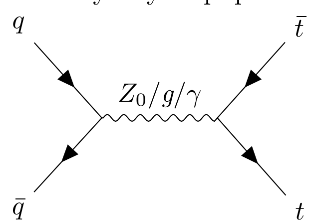
# 1 Abstract

High Energy Particle Physics deals with the behaviour and dynamics of particles in extreme energy collisions. Electrons are fundamental particles and are classified as *leptons*. Electrons carry electric charge and thus interact with other particles through a intermediary photon. Quarks are non-leptonic fermions, which in addition to electric charge carry the color charge and thus can interact through and intermediary photon, weak boson, or a gluon. By far the quark physics is the most complicated since they tend to hadronize (form larger structures or hadrons to hide their color charges, a phenomenon called "color confinement"). The Top quark, the subject of study of this project is a third generation quark and is also the heaviest of all quarks at around $m = 173\ GeV$. Top physics has been said to be crucial to understanding the complex workings of Quantum Chromodynamics. In this project we will study the angular correlations of two outgoing top quarks, with the incoming protons.


# 2 Introduction

The following are some of the may ways top quarks can be produced.




Since the initial momentum of the incoming quarks is not known (it is determined by the Parton Distribution Function), we cannot construct the centre of mass frame of this process. All we know is the following:

- Sum of the momenta of all the incoming quarks and gluons is exactly zero, since the process in in the centre of mass frame of the protons.
- Both the protons contain three quarks and many gluons, but only two of those particles come together to collide and form a pair of top quarks.
- The colliding particles can be gluons, up quarks, down quarks, they can come from the same proton or different protons. If the colliding particles exactly balance each other's color, then the process can also proceed through a $\gamma$ or a $Z_0$ Boson.
- The Z momenta of the incoming particles is large enough to ignore their transverse momenta.
- The final four momenta of the Top Quarks.

The exact mechanism of interaction of the fermions is governed by their fermionic fields which appear as solutions to the Dirac Equation, the description of which is beyond the scope of this project. There is significant theoretical literature on this subject [5], and given that the exact theoretical treatment is not only very complicated but also not clearly understood by the author, we will take a purely phenomenological approach.

## 2.1 About Lorentz Boost

We are working at the TeV scale, the mass of the heaviest particle under study is of the order of a few GeVs. From teh relativistic relation $E = \sqrt{p^2 + m^2}$, we can tell that $p = \sqrt{E^2 - m^2}$ or if the mass is small, (in our case it is about $10^{-3}$ times smaller) we can approximate $p \approx E$, in that case we see that the velocity of the particle $v = c \times (\frac{E/mc^2}{1+E/mc^2})$. For electrons, $E/mc^2$ can be found as $\approx 2\ TeV/0.511\ MeV = 3.91 \times 10^6$, hence the correction factor is roughly 0.99999974424. So the velocities our electrons will be tested at are extremely close to the speed of light. Thus we will need to take Lorentz boost into account.

The general Lorentz Boost along X direction is given by the following equations:

$$t' = \gamma(t - \beta x)$$
$$x' = \gamma(x - \beta t)$$
$$y' = y \quad z' = z$$

But remember that this transformation is valid for any four vector, so defined to be invariant under a Lorentz Boost, such as $(E, \vec{p})$ as well.

### 2.1.1 Lorentz Boost in Transverse Direction

Coming back to our specific problem, we first try to set up a working coordinate system. Let the $z$ axis coincide with the Particle Collider Beam. Then the 14 $TeV$ energy that we are talking about gets split equally between two colliding protons (as we are in the centre of mass frame) and both are oriented along the $z$ axis, opposite in direction. The first proton (henceforth $\pi_1$) is moving with $p_x = p_y = 0;\ p_z = 7\ TeV$, while the second proton is moving with $p_x = p_y = 0;\ p_z = -7\ TeV$ in the centre of mass frame.

Since the mass of the Proton is ($= 938\ MeV$) almost ten millions times smaller than it's energy, the quarks and the gluons inside the proton will be rattling around with a very uncertain momenta themselves. However what is certain is that if we add all their X and Y momenta, we will land up at zero. The total energy of the proton is split amongst it's constituents in a very well defined way, which is given by the Parton Distribution Function, which can be applied to "see" the insides of a hadronic structure [3]. Then the basic idea reduces to the following, since the momentum transverse to $z$ axis is zero, it must be small enough with respect to $p_z$ for any parton within the protons, that the Lorentz boost in that direcion can be safely ignored.

In fact we can use the Uncertainty Principle to estimate the minimum Transverse Momentum for any parton, and then stay several orders above it. For instance, at LHC the impact parameter is in orders of 300 $\mu m$ [4]. This means the uncertainty in momentum is at least $1.76 \times 10^{-31}$, and since we know the expectation value to be zero, we may safely ignore this in the transverse direction. However we will have to write it out for the $z$ axis, which we do in the following way.

### 2.1.2 Lorentz Boost along the beam

Lets say particle 1 and 2 have been found, and we have to find a $\beta$ *and* $\gamma$ such that it takes us from the current reference frame to their centre of mass frame. Let their four momenta be $(E_1, \vec{p_1})$ *and* $(E_2, \vec{p_2})$ in the current frame and $(E_{1cm}, \vec{p_{1cm}})$ *and* $(E_{2cm}, \vec{p_{2cm}})$ in the centre of mass frame. Then we have the following relations (dropping the vector momentum for $z$ axis momentum):

$$p_{1cm} = \gamma(p_1 - \beta E_1) \tag{1}$$

$$p_{2cm} = \gamma(p_2 - \beta E_2) \tag{2}$$

$$p_{1cm} + p_{2cm} = 0 \tag{3}$$

By using the above three equations we find that

$$\beta = -\frac{p_1 + p_2}{E_1 + E_2} \; and \; \gamma = \frac{1}{\sqrt{1 - \beta^2}} \tag{4}$$

while keeping in mind that $p_1$ and $p_2$ are the $z$ axis momenta, not the magnitudes of the three momenta.

### 2.1.3 Boosting back angles

Now we now how to boost particles, then calculating the inverse Lorentz Boost is simple, we just reverse the direction of $\beta$. BUt how do we calculate the angle in the centre of mass frame? We must now consider three particles, two of them either outgong or incoming, to reconstruct the centre of mass frame, and one more to calculate the angle against. Let these three particles be an incoming quark ($u$), and the outgoing top quarks ($t$ and $\bar{t}$). Let their four momenta be $(E_u,\ px_u,\ py_u,\ pz_u)$, $(E_t,\ px_t,\ py_t,\ pz_t)$ and $(E_{\bar{t}},\ px_{\bar{t}},\ py_{\bar{t}},\ pz_{\bar{t}})$ respectively. We have already established that our boost is *mostly* along $z$ axis, hence the $x$ and $y$ components of these four vectors will remain unchanged, while their time and $z$ components will mix according to the rules of the Lorentz Transform.

For any given event, the $\beta$ that takes us from the lab frame to the centre of mass frame of the top quarks (which will the same as the centre of mass frame of the incoming quarks), is given by equation 4 where the momentum has been replaced by the $z$ momenta of the top quarks. Then we reverse the sign of $\beta$, and write our boosted back momenta for the $u$ and $t$ quarks:

$$\beta = -\frac{pz_{\bar{t}} + pz_t}{E_{\bar{t}} + E_t} \tag{5}$$

$$pz'_u = \gamma(pz_u + \beta E_u) \tag{6}$$

$$pz'_t = \gamma(pz_t + \beta E_t) \tag{7}$$

Then the angle of interest ($= \theta_{13}$) is given by:

$$cos(\theta_{13}) = \frac{\vec{p_u} \cdot \vec{p_t}}{|\vec{p_u}||\vec{p_t}|}$$

$$= \frac{px_u px_t + py_u py_t + pz'_u pz'_t}{\sqrt{px_u px_u + py_u py_u + pz'_u pz'_u}\sqrt{px_t px_t + py_t py_t + pz'_t pz'_t}}$$

## 2.2 The Challenge in Hadronic Colliders

Hadrons are composite particles made out of a combination of the quarks and gluons, carrying and interacting with color charges. They participate in strong interaction, often out shadowing any other channels in a given process. Since the color charge comes in three types, and the gluon which carries the force is also charged, (unlike the photon which carries the force in eletromagnetism but isn't charged), the theoretical model of the strong interaction is very complicated.

The Large Hadron Collider is a proton-proton collider. It is the world's largest and the most powerful particle collider [2]. It first started up on 10 September 2008, and remains the latest addition to CERN's accelerator complex. The LHC consists of a 27-kilometre ring of superconducting magnets with a number of accelerating structures to boost the energy of the particles along the way. It is almost a hundred meters underground and 27 kilometers in diameter. It has four detectors which are run by various organisations. They are ALICE, LHCb, CMS, ATLAS. [1]

### 2.2.1 Hadrons in the Standard Model

The standard model has been classified into two kinds of particles, particles whose wave function is anti-symmetric (fermions) and whose wave function is symmetric (bosons) with respect to particle exchange. The fermions are of two types, the ones that participate in the strong interaction (quarks) and the ones that do not (leptons). Electron is a type of lepton for example. However the key fact is that the gluons also carry color. Let us take a naturally occurring hadron, made up of two quarks lets say $d\bar{d}$ (ignore for a moment that it might annihilate itself). This hadron would be chargeless and could be spinless. While each quark could carry any and all of the three colors, if you were to check, all such particles ever produced would be seen to be colorless. The quarks are constantly producing, passing around and absorbing gluons, thus producing the strong interaction which keeps them together. However if we wish to see some color, we would give it some energy, stretching their strong bond. In this process, the quarks themseleves with get father and father apart and the gluons in between will get more and more energetic. The ultimate goal is to separate the quarks and observe their colors. However the strong force is strong, so by the time the quarks are far apart enough for us to see, something interesting has happened. The gluon bond in between has been broken. We don't see any color yet, so we probe further, and notice the hadron has split into two different particles, since the application of more energy sends it's two pieces flying. Suddenly both of the pieces have gained electric charge, and the rest mass of both the pieces has increased by roughly half. This is strange, we don't expect the quarks to gain mass and charge when separated, they carry fractional charge. Looking even more closely we see that both the particles are actually new hadrons, one being $d\bar{u}$ carrying a -1 charge, and the other one is $\bar{d}u$ carrying a +1 charge. We have been successful in separating the $d$ and the $\bar{d}$, but in the process we have created a new quark and anti-quark pair of $u$ and $\bar{u}$, effectively reducing the energy of the system and saving the gluon bonds in the new hadrons. After all this work we realise that we still see no color. This principle is called $Color\ Confinement$. The strong force is so strong that in order to break it's bonds we need to supply so much energy, that the gluons create a quark anti-quark pair instead of just breaking down. Thus in real situations, we are never able to see individual colored particles at all.

### 2.2.2 The Parton Distribution Function

Above is an example Parton Distribution Function, which are determined empirically and often tested enough to agree with experiment rigorously. The $x$ axis shows the fraction of the energy of the overall system, while the $y$ axis shows the probability.

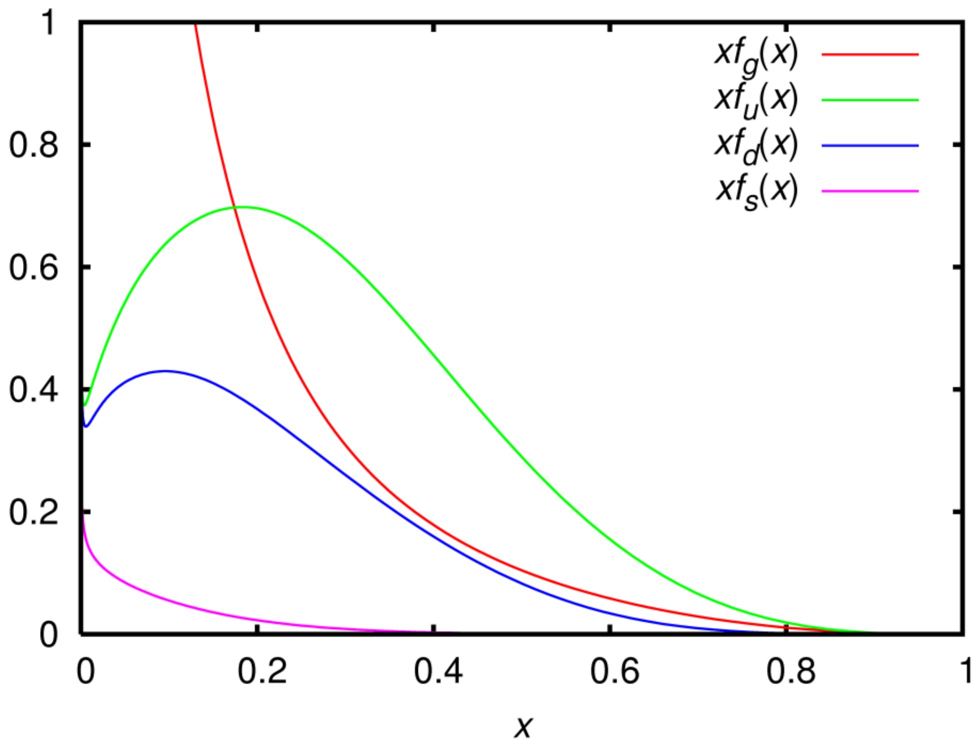



Figure 1: The CTEQ6 parton distribution functions in the MS renormalization scheme and Q = 2 GeV for gluons (red), up (green), down (blue), and strange (violet) quarks. Plotted is the product of longitudinal momentum fraction x and the distribution functions f versus x. [6]

The Parton Distribution Function is the function which tells you how longitudinal energy is distributed in a hadron among it's constituent particles. From the graph it is clear that around 13% energy in the hadron is always carried by every gluon, while the two up quark is most probably carrying around 21% and the down quark is most probably carrying another 10% energy.

#### 2.2.3 Centre of Mass Frame in Hadronic Colliders and Leptonic Colliders

The above context of Color Confinement and Parton Distribution means that when we are studying the collision between two protons, we don't really know the momenta distributions of the quarks within. Since the mass of the proton is only 931 $MeV$, which is negligible in comparison to the 14 $TeV$ the LHC will soon be operating at, the protons themselves don't participate in the process at all. In fact the bounds of the proton are no longer a thing at such a high energy domain, but it is the quarks that are scattered off of each other.

In case of leptonic colliders such as any electron positron collider, we know the exact momenta (within Heisenberg's Uncertainty Limit) of the electrons and the positrons, so we can simply move our calculation into the centre of mass frame. However in case of the LHC, the quarks participating in the process mght be going in any direction permitted by the Parton Distribution Functions, which means the reconstruction of the Centre of Mass is impossible. Also notable is the fact that it might not even be the quarks reacting, but it could be some stray gluon from one of the protons which produces a quark anti-quark pair one of which then generates the process of interest. Thus in order to analyse the dynamics of such a process, we not only need better tools, but also a large amount of data to reach an acceptable significance level.

The entire process is made even more complex because in reality there might be several other side reactions that might or might not perturb our process of interest. Also significant are the initial state and the final state radiations, which are radiations from the particles just before and after the process but do not alter the process. The the top quark (the subject of our study) is produced in some direction but then it radiates a photon or a gluon and gets scattered, it will change it's momentum leading to false readings. Our theories must account for all of these types of radiations and scatterings and energy distributions, everything while predicting the observable detector readings, in which the Standard Model of Particle Physics has been quite successful so far.

# 3 Method

We will use a particle simulator called PYTHIA [7] (link). It is a well accepted and tested particle collider simulator, it can be used to write various particle collision settings and test models including and beyond the Standard Model.

Thus I wrote a script that instructs PYTHIA to do the following things:

1. Take the Centre of Mass Energy (henceforth **E**) as an argument from command line,
2. Initialise it's libraries and set collision particles to two protons,
3. Consider $\gamma$ channel, $Z_0$ channel and $g$ channel mediation of quarks,
4. Disable all quark decays and enable only the process corresponding to $f\bar{f} \rightarrow t\bar{t}$,
5. Disable top decay and hadronization,
6. Calculate, boost back (see previous section) and print out the required $cos(\theta_{13})$ to a file corresponding to one particular value of **E**.

The script written has a loop that will calculate $N = 10000$ such collisions and dump their output angles into a file. At this point we have all the angles we need to construct a histogram in the file. The problem is that it has been calculated at just one value of **E**. So I wrote a bash script that simply called the PYTHIA code with various values of **E** (sent as the command line parameter).

All of this code and data is available at (a GitHub repository).

I then wrote a bash script to call the PYTHIA code almost 140 times, to simulate various energy levels. This created 140 different files with the output angles. Since reading from files is very slow, I zipped them into a Python Binary called the "Z.bin.npy" (after reading through the files once) available over at the repository.

Then I simply used the MatPlotLib python package to visualise the results. This generates 150 different histograms, with $cos(\theta_{13})$ on the $x$ axis and number of events on the $y$ axis.

## 4 Results

The histograms our the results. They demonstrate the probability of the top quarks to form a given angle in a proton proton collision. For example, in Figure 1a. below, the bars at $cos(\theta_{13}) = 0$, are approximately around the value of 250. Since we know we simulated $N = 10000$ events, roughly 250 of them landed at $cos(\theta_{13}) = 0$. Hence for $\theta_{13} = \pi/2$, the probability is roughly $\frac{250}{10000} = 0.025$. We must also account for the width of the bars in our plot(which are 35 in number). The angular resolution can be described by:

$$-\frac{1}{35} \leq cos(\theta_{13}) \leq \frac{1}{35}$$
$$\implies 91.64° \leq \theta_{13} \leq 88.36°$$

Hence formally speaking, the probability that the top quark bounces at an angle in the range $[88.36°,\ 91.64°]$ is 0.025. Note that this angle is only the angle from the incoming quark's direction, not the angle between the the beam axis the top quark (which is often what is actually measured). Since the transverse momentum of the incoming quarks is negligible, $\theta_{13}$ is approximately the same as the angle between the top quark and the beam axis.

The next feature of prominence is the curvature of the histograms, which is a constant at all tested values of **E**. According to theory [5], two fermions going to two fermions in the final state always produces an angular correlation of the form $1 + cos^2(\theta_{13})$. This is suitably realised in all the produced plots, for all tested values of **E**. This means that regardless of the propagator involved in the top quark production (it could any of $\gamma,\ g,\ Z_0$), the top quarks always tend to go back to back along the direction of the beam axis. This is a significant fact and can be used to distinguish particles in the final state based upon their spin. For example, a scalar particle in the final state is expected to produce a curve of the form $1 - cos^2(\theta_{13})$ and a vector particle can be characterized in a similar way.

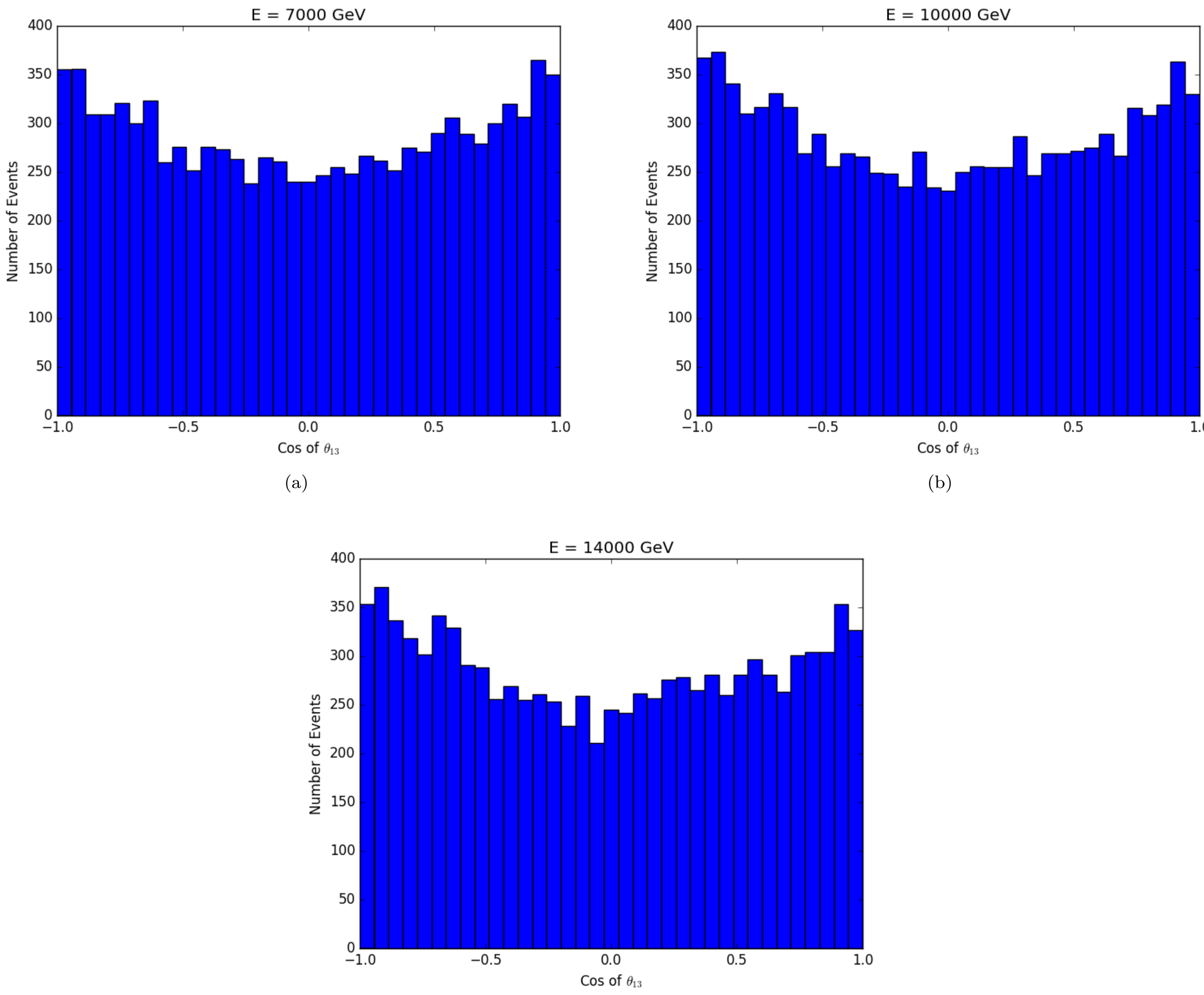



Figure 2: Angular Correlations at $\mathbf{E}$ = 7 TeV, 10 TeV and 14 TeV

# References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